On the Maximum Observed Wind Speed in a Randomly Sampled Hurricane

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ABSTRACT

There is considerable interest in detecting a long-term trend in hurricane intensity possibly related to large-scale ocean warming. This effort is complicated by the paucity of wind speed measurements for hurricanes occurring in the early part of the observational record. Here, results are presented regarding the maximum observed wind speed in a sparsely randomly sampled hurricane based on a model of the evolution of wind speed over the lifetime of a hurricane.

1. Introduction

There is considerable interest in detecting and understanding historical variations in the intensity of hurricanes as measured, for example, by maximum wind speed. This interest stems in part from a possible connection between hurricane intensity and large-scale climate change (Emanuel 2005). A potentially serious problem in identifying such variation is that only a few chance wind speed measurements may be available for hurricanes in the earliest part of the observational record (Landsea et al. 2004a). The purpose of this note is to describe some general statistical results concerning the maximum observed wind speed in a hurricane that is sparsely observed at random times during its lifetime. These general results are then specialized to a model of the evolution of wind speed within a hurricane based on Emanuel (2000).

2. The basic result

In this section, the basic statistical result is outlined. This result is specialized to a particular case in section 3.

Consider a hurricane with lifetime $(0, T)$. Let $v(t)$ be the maximum wind speed of this hurricane at time t. Suppose that this hurricane is observed at random times $t_1, t_2, \ldots, t_n$ during its lifetime and let the random variables $W_1, W_2, \ldots, W_n$ be the maximum wind speeds observed at these times. To begin with, assume that the error in these observations is negligible, so that $W_j = v(t_j)$. This assumption is relaxed later.

The distribution function of a randomly observed wind speed W is given by

$$F(w) = 1 - \frac{T(w)}{T}, \tag{1}$$

where $T(w)$ is the total time during which wind speed exceeds w. The corresponding probability density function (pdf) is given by

$$f(w) = -\frac{T'(w)}{T}. \tag{2}$$

The support of this pdf—that is, the values of v over which $f(w)$ is positive—has an upper bound at the maximum value $v_{\max}$ of $v(t)$ over the interval $(0, T)$.

Let $W_{(1)} < W_{(2)} < \cdots < W_{(n)}$ be the observed wind speeds ordered from smallest to largest, so that $W_{(n)}$ is the maximum observed wind speed. It is a standard result that the pdf of $W_{(n)}$ is

$$g(w) = nf(w)F^{n-1}(w) \tag{3}$$

(David and Nagaraja 2003). Clearly, the support of $g(w)$ also has an upper bound at $v_{\max}$. Beyond that, the behavior of $g(w)$ depends on $F(w)$, which depends in turn on $v(t)$. For large n, the distribution of $W_{(n)}$ will converge

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to the Weibull extreme value distribution, which is the only extreme value distribution with finite support. However, the situation of interest here is when n is small.

This basic model can be extended to allow for error in observed wind speed. For example, suppose now that observed wind speed W_j is given by the sum

$$W_j = v(t_j) + \varepsilon_j \tag{4}$$

of true wind speed at time t_j and a normal observation error with mean 0 and variance σ^2. Under this model, the distribution function of a randomly observed wind speed is given by the convolution

$$F_e(w) = \int_0^\infty \Phi\left[\frac{(w-v)}{\sigma}\right] f(v)\, dv, \tag{5}$$

where Φ is the standard normal distribution function and $f(v)$ is given in (2). Here and below, the subscript e is used to indicate results for the error model in (4). The corresponding pdf is

$$f_e(w) = \frac{1}{\sigma}\int_0^\infty \varphi\left[\frac{(w-v)}{\sigma}\right] f(v)\, dv, \tag{6}$$

where φ is the standard normal pdf. Assuming that the errors in the observations are independent, the pdf $g_e(w)$ of $W_{(n)}$ has the same form as (3) with f and F replaced by f_e and F_e, respectively. In this case, the support of $W_{(n)}$ is unbounded; therefore, its limiting distribution is no longer Weibull.

3. A special case

Emanuel (2000) showed that the evolution of intensity in Atlantic Ocean and western North Pacific Ocean hurricanes whose maximum intensity was not limited by declining potential energy exhibited remarkable regularity, with wind speed increasing linearly by around 12 m s^{-1} day^{-1} to maximum wind speed, followed by a linear decay of around 8 m s^{-1} day^{-1}. Based on Emanuel's result, suppose that

$$\begin{aligned} v(t) &= v_0 + \beta t & 0 \le t \le t_{\max} \\ &\quad v_{\max} - \gamma(t - t_{\max}) & t_{\max} < t \le T \end{aligned} \tag{7}$$

with β and $\gamma > 0$, where $v_{\max} = v_0 + \beta t_{\max}$ and $T = t_{\max} + (v_{\max} - v_0)/\gamma$. Under this model, the lifetime of a hurricane begins when wind speed reaches v_0. Wind speed then increases linearly at rate β until reaching a peak of $v_{\max}$ before declining linearly at rate γ until it again reaches v_0 at time T.

It is straightforward to show that, in the absence of observation error, a randomly observed wind speed W under this model has a uniform distribution over the interval $(v_0, v_{\max})$ with distribution function

$$F(w) = \frac{w - v_0}{v_{\max} - v_0} \quad v_0 \le w \le v_{\max} \tag{8}$$

and pdf

$$f(w) = \frac{1}{v_{\max} - v_0} \quad v_0 \le w \le v_{\max}. \tag{9}$$

It follows from (3) that the pdf of $W_{(n)}$ is

$$g(w) = n\frac{(w - v_0)^{n-1}}{(v_{\max} - v_0)^n}. \tag{10}$$

For $n > 1$, $g(w)$ increases monotonically with w, becoming increasingly concave as n increases. The expected value of $W_{(n)}$ is given by

$$E[W_{(n)}] = v_0 + \frac{n}{n+1}(v_{\max} - v_0) < v_{\max}. \tag{11}$$

The relative underestimation bias in using $W_{(n)}$ as an estimate of $v_{\max}$ is

$$\frac{\{v_{\max} - E[W_{(n)}]\}}{v_{\max}} = \frac{(1 - v_0/v_{\max})}{(n+1)}. \tag{12}$$

So, for example, if $v_0/v_{\max} = 0.25$, then the relative underestimation bias is around 38% for $n = 1$ and 13% for $n = 5$.

Turning to the case in which wind speed is observed with normal error, it is straightforward to show that, for the wind speed model in (7), the pdf of a randomly observed wind speed W is given by

$$f_e(w) = \frac{\Phi[(w - v_0)/\sigma] - \Phi[(w - v_{\max})/\sigma]}{v_{\max} - v_0}. \tag{13}$$

No closed form expressions for $F_e(w)$, $g_e(w)$, or $E_e[W_{(n)}]$ are available, but it is straightforward to evaluate these numerically. For example, Fig. 1 shows $g_e(w)$ for the case $v_{\max} = 1$, $v_0 = 0.25$, $n = 4$, and $\sigma = 0.1$. For comparison, Fig. 1 also shows $g(w)$ for the same values of $v_{\max}$, v_0, and n. In Fig. 2, $E_e[W_{(n)}]$ is plotted against n for these values of $v_{\max}$, v_0, and σ. Again, for comparison, Fig. 2 also shows $E[V_{(n)}]$ for the same values of $v_{\max}$ and v_0. It is notable that, by opening the possibility of overestimation, the presence of measurement error actually reduces estimation bias.

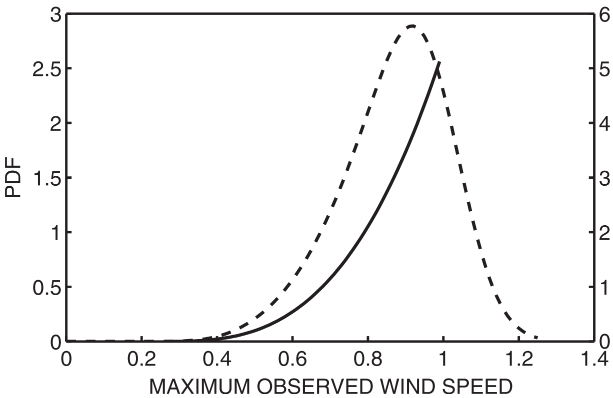

FIG. 1. The pdf of maximum observed wind speed for $v_{\max} = 1$, $v_0 = 0.25$, $n = 4$, and $\sigma = 0$ (solid) and $\sigma = 0.1$ (dashed).

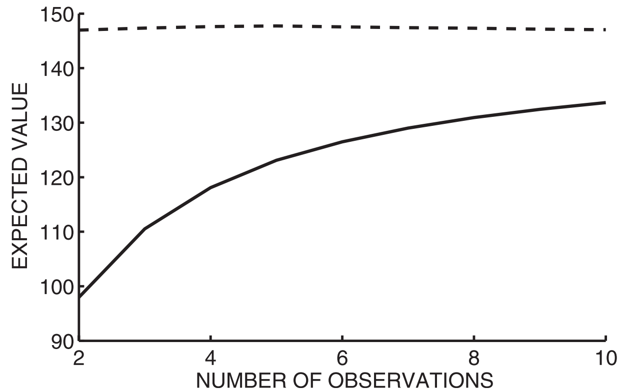

FIG. 3. Expected value of maximum observed wind speed (solid) and expected value of reconstructed maximum wind speed (dashed) vs n for Hurricane Andrew.

4. A practical application

The theoretical results presented in the previous section provide the basis for a rough practical method to correct for underestimation bias. The expression in (11) can be rearranged as

$$v_{\max} = \frac{n+1}{n}E[W_{(n)}] - \frac{1}{n}v_0 \cong \frac{n+1}{n}E[W_{(n)}]. \tag{14}$$

Thus, a rough correction for underestimation bias can be made by inflating the observed value of $W_{(n)}$ by the factor of $(n + 1)/n$, leading to the estimator $\hat{v}_{\max} = W_{(n)}(n + 1)/n$. This requires knowledge of only n and $W_{(n)}$.

A simulation experiment based on the reanalyzed National Hurricane Center North Atlantic basin best-track hurricane database (known as "HURDAT") wind speed measurements for Hurricane Andrew reported in Table 1 of Landsea et al. (2004b) was conducted to assess the performance of $\hat{v}_{\max}$. For this analysis, v_0 was taken to be 35 kt (1 kt $\approx$ 0.5 m s^{-1}). The data consist of 39 6-hourly wind speed measurements covering a 228-h period with a maximum of 150 kt. These data were taken to represent the true evolution of maximum wind speed for Hurricane Andrew, with values between observation times reconstructed by linear interpolation. This profile is shown in Landsea et al. (2004b, their Fig. 4) and also in Emanuel (1999, his Fig. 2).

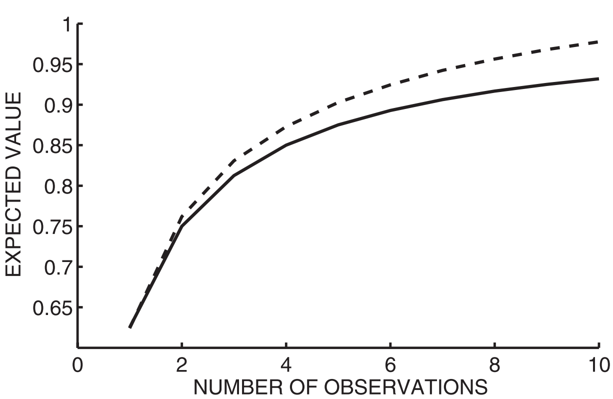

FIG. 2. Expected value of maximum observed wind speed vs n for $v_{\max} = 1$, $v_0 = 0.25$, and $\sigma = 0$ (solid) and $\sigma = 0.1$ (dashed).

The simulation experiment proceeded in the following way. For each value of n between 2 and 10, wind speeds were sampled at n random times from this profile, and both the maximum $W_{(n)}$ of these n wind speeds and the estimate $\hat{v}_{\max}$ based on it were recorded. The procedure was repeated a total of 10 000 times. In Fig. 3, the averages of the 10 000 values of $W_{(n)}$ and $\hat{v}_{\max}$ simulated in this way are plotted against n. In this case, $\hat{v}_{\max}$ performed very well, on average, underestimating $v_{\max}$ by only around 3 kt for n between 2 and 10. In contrast, on average, $W_{(n)}$ underestimated $v_{\max}$ by more than 50 kt when $n = 2$ and almost 20 kt when $n = 10$.

The experiment was repeated for a small number of other hurricanes listed in Table 1. Wind speed data for these hurricanes were available online at the HURDAT Internet site, and the corresponding wind speed profiles are shown in Emanuel (1999). Table 1 reports the value of $v_{\max}$ and the average values over 10 000 simulated

TABLE 1. Values of $v_{\max}$ and averages of more than 10 000 simulations of $W_{(n)}$ and $\hat{v}_{\max}$ for $n = 3$ for selected North Atlantic hurricanes. All wind speeds are reported in knots.

Name	Year	$v_{\max}$	Avg $W_{(n)}$	Avg $\hat{v}_{\max}$
Gloria	1985	125	87.5	116.7
Gilbert	1988	160	123.9	165.2
Dean	1989	90	79.4	105.9
Hugo	1989	160	129.6	172.9
Andrew	1992	150	110.1	147.1
Opal	1995	130	87.5	116.7

random wind speed samples of $W_{(n)}$ and $\hat{v}_{\max}$ for $n = 3$. With the exception of Hurricane Dean, the absolute relative bias of $\hat{v}_{\max}$ is less than 10%, with $v_{\max}$ underestimated in half the cases and overestimated in the other half. In contrast, by necessity, $W_{(n)}$ always underestimates $v_{\max}$ with a relative underestimation bias of up to 32%. It is only for Hurricane Dean that the absolute bias of $W_{(n)}$ is smaller than that of $\hat{v}_{\max}$. The reason is that the wind speed profile for this hurricane is far from the model in (7) and, in particular, exhibits a plateau just below its peak. In overall terms, given its extreme simplicity, $\hat{v}_{\max}$ appears to perform well at correcting the underestimation bias of $W_{(n)}$.

5. Discussion

The purpose of this note has been to outline and illustrate a statistical formalism for exploring the effect of sparse random sampling on the maximum observed wind speed in a hurricane. The model considered here is clearly stylized and can be extended in a number of ways. In particular, realism could be gained through an explicitly spatial model of hurricane wind fields and their sampling. In some situations, it may be reasonable to assume that observers seek to avoid the highest winds. This would have the effect of exacerbating underestimation bias. On the wind speed side, the model for $v(t)$ in (7) can be extended to include a random component, so that $v_{\max}$ is itself a random variable.

The focus here has been on maximum wind speed. In some situations, interest centers on a function of it. For example, the power dissipation index of Emanuel (2005) depends on the cube of maximum wind speed. The underestimation bias in estimating $v_{\max}^3$ by $W_{(n)}^3$ is worse than that in estimating $v_{\max}$ by $W_{(n)}$. For example, in the absence of measurement error, when $v_0/v_{\max} = 0.25$, the relative bias in estimating $v_{\max}^3$ is 0.51 when $n = 2$ and 0.30 when $n = 5$. As reported earlier, the corresponding values in estimating $v_{\max}$ are 0.38 and 0.13.

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